Firm Formation with Heterogeneous Management and Labor Skills

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ABSTRACT. When both management skills and labor skills differ in the population, the nature of firm formation can differ radically from the equilibrium we are used to dealing with: The best potential managers could end up as wage workers. This happens when managerial skills are positively correlated with working skills, and there is some empirical evidence that this is indeed the case.

1. Introduction

The study of small business forces one to describe environments where small firms can coexist with large firms. This almost immediately leads one to specify ways in which firms differ. Models of this type suppose that firms vary in their endowments of a fixed factor in inelastic supply, and where in equilibrium all but the marginal firms can earn Ricardian rents that stem from their relative abundance in the fixed factor in question. In these situations firm size differences originate in differences in their endowments of the fixed factor. A partial equilibrium model of one industry is in Panzar and Willig (1978), and a general equilibrium treatment is in Lucas (1978).[1] To study the birth, growth, and death of firms, it then becomes useful to assume that the firm does not know its endowment of the fixed factor, and that it learns with experience about its suitability to a particular industry. This was done by Jovanovic (1982).

In this class of models, the firms' workers are alike. Or, the firm's labor input can be thought of as measured in "efficiency units", and in the market for these efficiency units, the firm behaves as a price-taker. Treating labor in this way means that a worker with y efficiency units of labor is the productive equivalent of two workers with $y/2$ units each, and such a treatment was first implemented empirically by Krueger (1968) and Welch (1969). A distribution of productivities in the workforce then induces a distribution of earnings. To study the evolution of earnings, it again became useful to assume that the worker and his employer do not know the worker's endowment of efficiency units, and that they both learn about it with experience. This was done by Harris and Holmstrom (1982).

The modelling of heterogeneous management and heterogeneous labor skills has therefore proceeded in parallel fashion. In this paper I will merge the two approaches, but I will limit the analysis to the situation where every person's skill endowment — be it as a worker or as a manager — is common knowledge. Formally, the paper generalizes the Lucas (1978) model of entrepreneurial choice by allowing for heterogeneous quality of labor. The resulting structure is a variant of the two-skill Roy (1951) model, which has been much discussed and extended, most recently by Heckman and Honoré (1990). In contrast to this literature, the model treats the prices of the two skills as endogenous, but is otherwise a straightforward application of the selection principles that Roy discussed.[2]

Labor incomes display a lot of variability over workers, so that this extension of Lucas' model certainly adds realism. Understanding the way that taxation (progressive income taxation in particular) affects entrepreneurial choice certainly demands that the heterogeneity of both entrepreneurial and non-entrepreneurial incomes be recognized. Moreover, adding heterogeneity in labor skills can

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radically change the nature of equilibrium: If the best managers are also the best workers, the economy's firms may end up being run by those least equipped to do so. In addition, the degree of correlation between managerial and working skills will significantly affect the distribution of income and the income differential between managers and workers.

Certain properties of the model do not, however, depend on there being differences in labor *or* in management skills. I shall develop these in the next section, and then turn to the properties of the more general structure. In the concluding section I will comment on some possible extensions of this model.

2. The simplest equilibrium model of firm formation: identical agents

I will begin with the simplest possible equilibrium model of firm formation where there are two homogeneous factors of production: capital, k, and labor, ℓ, and in which management skills are identical in the population. This is the model that I think we all have in the back of our minds when discussing the basics of firm formation, and yet I have not seen it spelled out anywhere.

In an economy with one consumption good, let $F(k, \ell)$ be a firm's output if it hires inputs k and ℓ. Let there be L identical agents each of whom has a unit of labor, and an economy-wide capital stock K. This is Lucas's setup, but with identical agents.[3]

Let w be the wage and r the rental on capital. Each agent can either run his own firm, or work for someone else. In equilibrium, entrepreneurial income must equal the wage:

$$\max_{k,\ell} \{F(k, \ell) - rk - w\ell\} = w. \tag{1}$$

Optimal employment of factors means that their marginal products must equal their prices:

$$F_k = r, \text{ and } F_\ell = w.$$

Let m be the number of managers so that the capital stock per manager is K/m, and (since working and managing exhaust the possible uses of one's time) the number of workers per manager is $(L - m)/m$. Thus in equilibrium, condition (1) reads

$$F\left(\frac{K}{m}, \frac{L-m}{m}\right) - \frac{K}{m} F_k\left(\frac{K}{m}, \frac{L-m}{m}\right) -$$

$$- \left(\frac{L-m}{m}\right) F_\ell\left(\frac{K}{m}, \frac{L-m}{m}\right)$$

$$= F_\ell\left(\frac{K}{m}, \frac{L-m}{m}\right).$$

Rearrangement and cancellation lead to

$$F\left(\frac{K}{m}, \frac{L}{m} - 1\right) - \frac{K}{m} F_k\left(\frac{K}{m}, \frac{L}{m} - 1\right) -$$

$$- \frac{L}{m} F_\ell\left(\frac{K}{m}, \frac{L}{m} - 1\right) = 0. \tag{2}$$

This condition yields the equilibrium number of managers m, conditional on the economy-wide endowment of capital and labor. Moreover, the equilibrium allocation of people to management and wage-work maximizes the value of aggregate output, in the sense that m solves the problem

$$\max_m \left\{ mF\left(\frac{K}{m}, \frac{L-m}{m}\right)\right\}, \tag{3}$$

subject to $0 \leqslant m \leqslant L$. This is seen immediately if one differentiates (3) with respect to m; that is, the first-order condition to the problem in eq. (3) is just equation (2). This optimality result is not surprising since the economy has no monopoly, no external effects and no public goods, and it holds, of course, in Lucas's model as well.

We now turn to the time-series behavior of firm size, $(L - m)/m$. Differentiating (2) with respect to K reproduces Lucas's result that if the elasticity of substitution between capital and labor is less than one, an increase in the economy's stock of capital, K, will raise employment per firm by lowering m, which responds to K as follows:

$$\frac{dm}{dK} = \frac{m(KF_{kk} + LF_{k\ell})}{K^2F_{kk} + 2KLF_{k\ell} + L^2F_{\ell\ell}} < 0. \tag{4}$$

The denominator is positive from the second order conditions that guarantee an interior solution to the problem in equation (1). Lucas was

dealing with a production function that was a concave transformation of a linearly homogeneous production function. Linear homogeneity of F would imply[4] $dm/dK = 0$, but the concave transform readers dm/dK negative. The intuition for the result is that when labor and capital are not too substitutable, the increase in the economy's capital stock raises wages more than it raises managerial returns and therefore causes an increase in the number of people who choose to be wage-workers, and an increase in firm size.

Since an increase in K can be viewed as capital-augmenting technical change, we have proved

PROPOSITION 1: *If $KF_{kk} + LF_{k\ell} < 0$, capital-augmenting technical change lowers the number of firms and raises average firm size.*

Looking ahead to the development of the next few sections, it is useful to establish a proposition regarding Hicks neutral shifts in the production function. Write every firm's output as $sF(k, \ell)$, where s is an exogenous technical change parameter. Clearly, equation (2) holds at the same value of m for *all* values of s. So we have proved

PROPOSITION 2: *Hicks-neutral technical change does not affect m. A corollary of this result is that profits, $(F - (K/m)F_k - (L/m)F_\ell)$, wages, (F_ℓ), and rentals, (F_k), all change in the same proportion as s.*

If one chooses to view the business cycle as being driven by Hicks-neutral technology shocks, then this proposition states that entry and exit should not respond to cyclical shocks. There is some evidence in favor of this implication: entry and exit behavior of German firms seemed to be unresponsive to cyclical shocks. Using the Hodick-Prescott time-series decompostiion, Boeri and Kramer (1993) find that incumbents are associated with the cyclical components of the economy, while entrants and exits are more associated with the trend. To be sure, there is evidence from the U.S. in Nelson (1959, chapter 5) and from Japan in Goto (1981, figure 1), that entry by *diversifying* firms shows a distinct procyclicality. In the model, however, such entry should not be interpreted as the result of movement of individuals from wage work into management, because it probably reflects a reallocation of activities among existing managers.

3. The general case

People have two options: managing others, or working for others. A person's managerial ability is denoted by x and his wage-working ability is denoted by y. The population cumulative distribution of abilities to manage and to work is $\Phi(x, y)$. The total number of agents is normalized at unity.

A manager of ability x can produce output

$$q = xQ(h)$$

with h efficiency units of labor. We now drop the capital input since its presence would not alter the main insights. The Kruger-Welch-type assumption that only h enters the production function is restrictive: It implies that one worker is a multiple of another, so that if worker 1 is more productive in a firm than worker 2, then he is more productive in every firm.[5] Although this point should be looked into in future elaborations of this model, I will maintain the efficiency units assumption so as to keep the model simple.

Let w be the market price of labor per efficiency unit, denominated in output units. Then manager x's profit is

$$\pi(x; w) \equiv \max_{h \geq 0} \{xQ(h) - wh\}. \tag{5}$$

This expression is increasing in x and decreasing in w.[6] The maximization of (5) implies the first-order condition

$$xQ'(h) = w, \tag{6}$$

which yields the efficiency units employment function

$$h(x; w) = Q'^{-1}\left(\frac{w}{x}\right). \tag{7}$$

This is the demand function for efficiency units which is increasing in x and decreasing in w.

Selection into entrepreneurship is governed by the inequality

$$\pi(x; w) \geq wy. \tag{8}$$

If eq. (8) is met, agent (x, y) chooses to be an entrepreneur. If it fails, he chooses wage-work.

An *equilibrium* is a set A in R^2, and a real number w, such that

$$A = \{x, y \mid \pi(x; w) \geqslant wy\} \tag{E.1}$$

$$\int_A h(x; w)\, d\Phi(x, y) = \int_{\sim A} y\, d\Phi(x, y) \tag{E.2}$$

where $\pi(\cdot)$ and $h(\cdot)$ are defined in (5) and (6), and where $\sim A$ is the complement of the set A; the latter is the set of entrepreneurs.

Condition (E.1) insists that occupational selection is made optimally. Condition (E.2) states that the demand for efficiency units of labor, given by the left-hand side of (E.2), should equal their supply, given by the right-hand side.

The number of managers is $m \equiv \int_A d\Phi(x, y)$. Since the number of agents in the economy is normalized at unity, the number of workers is $1 - m$, and employment per firm is $(1 - m)/m$.

4. Analysis of equilibrium in two special cases

As expected, higher ability to manage leads to higher profits; by the envelope theorem,

$$\frac{\partial \pi}{\partial x} = Q(h(x; w)) > 0. \tag{9}$$

Moreover, since h is increasing in x, the profit function is convex in x:

$$\frac{\partial^2 \pi}{\partial x^2} = Q'[h(x; w)] \frac{\partial h}{\partial x} > 0. \tag{10}$$

Both of these properties, (9) and (10), would hold even if labor skills were homogeneous. The nature of equilibrium is easiest understood by first considering two special cases.

Case 1: Perfect positive correlation between x and y

The easiest case to grasp is one in which good entrepreneurs are invariably also good workers and conversely. That is, $y = \psi(x)$ for some strictly increasing function ψ. The equilibrium set of entrepreneurs then consists of those people whose x satisfies the inequality

$$\pi(x; w) \geqslant w\psi(x).$$

Denote the entrepreneurial ability of the marginal worker by z. That is, z satisfies

$$\pi(z; w) = w\psi(z).$$

Figure 1 illustrates the nature of equilibrium in the case where $\psi(x) = x$.

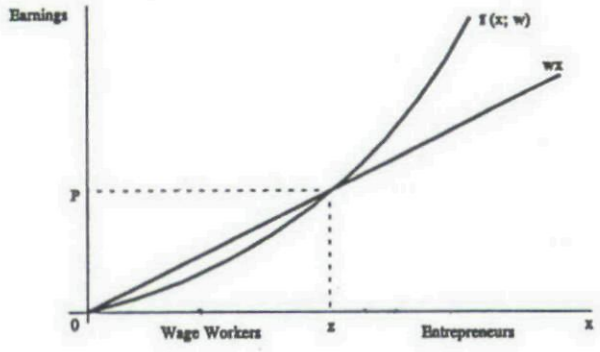

Fig. 1. Equilibrium selection when $x = y$ w.p.1.

In this case the best potential entrepreneurs are drawn into entrepreneurship. However, a perverse outcome is possible if $\psi(x)$ is sufficiently steep at higher levels of x; in this case the high x types would be drawn into wage-work. This possibility is illustrated in Figure 2. This situation arises in much of professional sport, where managers typically are paid less than many of the players that they manage. See Chapman and Southwick (1991), for instance.

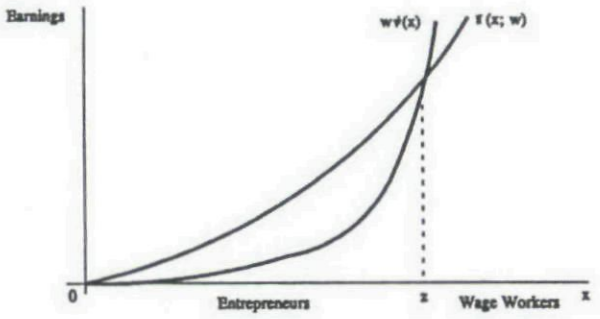

Fig. 2. The perverse case of a highly steep $\psi(x)$.

This is an equilibrium in which the worst managers end up doing the managing! In spite of this, in this atomless world without externalities or

income taxation, this outcome still maximizes aggregate output, as did Lucas's equilibrium. Of course, more complicated equilibria with multiple crossings of $w\psi$ and π are possible as well. Such perverse and complex equilibria could not arise in Lucas's model; there the best managers always end up doing the managing, and the worst managers end up in wage-work.

Broader empirical evidence overwhelmingly favors the assumption of positive correlation between x and y. People's observable characteristics such as their age, sex, and schooling, for the most part affect earnings in the same qualitative way: College graduates earn more whether they are wage workers or entrepreneurs, handicapped workers earn less, married people earn more, and so on.[7] Whether the situation is an extreme as in Figure 2, however, is not known.

Case 2: Perfect negative correlation between x and y

Here we again assume that $y = \psi(x)$, but that now ψ is a *decreasing* function. Here equilibrium is well behaved in the sense that the high x types end up being entrepreneurs, as they do in Lucas's model. This is illustrated in Figure 3.

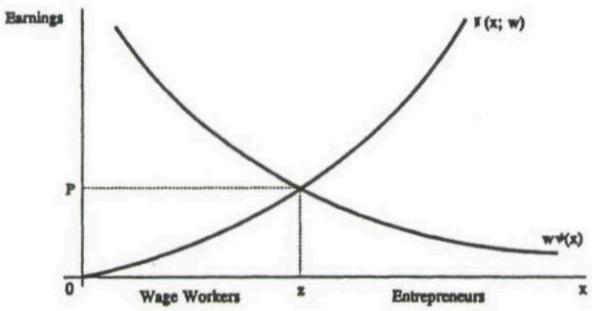

Fig. 3. Perfect negative correlation between x and y.

A comparison of the first and third figures shows that the gap between entrepreneurial incomes and labor incomes will be much smaller if x and y are negatively correlated than if they are positively correlated. In the latter case, the distribution of workers' earnings is in the interval $[0, p]$, while the distribution of entrepreneurial earnings is in the region above p. In the negatively correlated case, workers and entrepreneurs both earn at least p, and the marginal agent (z) has the global minimum earnings. This implication is, of course, testable with available micro data.

More generally, holding marginal distributions of talent constant, that is fixing $\int \Phi(dx, y)$ and $\int \Phi(x, dy)$, the lower is the correlation of x and y, the lower will be the variance of equilibrium incomes.[8]

5. Equilibrium in the general case

As in the simpler model of section 2, a general proposition holds with respect to Hicks-neutral technical change, technical change that leaves the marginal products of entrepreneurship and of labor unaffected relative to each other. The equilibrium wage will rise in proportion to the technical change. Let $Q(h) = s\hat{Q}(h)$ where s is a Solow residual-type shift-term. We then have the counterpart of Proposition 2:

PROPOSITION 3: *A change in s leads to an equiproportional change in w, and it has no effect on equilibrium allocations.*

Proof: Eq. (7) now reads $h(x; w) = \hat{Q}'^{-1}(w/sx)$ so that if w/s does not vary with s (as the Proposition claims) then $h(x; w)$ does not vary with s either. Hence if A is unchanged, so is total labor demand as well as total labor supply. That is (E.2) is met. All that remains is to show that A is unchanged. But this too is true because if w varies in proportion with s, so will π as defined in eq. (5). □

This proposition continues to hold if other inelastically supplied factors such as physical capital are added to the model, as was done in section 2. The prices of all inelastically supplied factors would vary in proportion to s, as was the case for the prices of both capital and labor. The counterpart of proposition 1 can not be proved here since I have excluded the capital input.

If we had a third activity, say "homework", where the payoff did not rise in proportion to s, then proposition 1 would fail. Indeed, labor force participation is mildly procyclical (Hall, 1980), and so market participation itself increases with s, but it is not known if the fraction of participants

choosing to be managers varies at all with the cycle.

6. Conclusion

When workers' skills as well as managers' skills differ, the selection of agents into entrepreneurship can differ radically from the one that Lucas described. The best talents may not flow into entrepreneurship, and this may indeed be the case in some fields such as professional sports. Also, the degree of correlation between the two types of skill will significantly affect the distribution of income in the two populations.

The static model is limited in some important ways. The selection equations assume that the agent can compare his two hypothetical incomes and choose the higher of the two. In fact the agent may be unsure about both incomes, and there may be learning about them over time. Jovanovic (1982) has a partial equilibrium model of learning about entrepreneurial skills, and Jovanovic and Moffit (1990) have a general equilibrium overlapping generations treatment of learning with more than one sector, but they do not consider entrepreneurship explicitly.[9] Kanbur (1979) allows people to be unsure about their entrepreneurial abilities, but his model is static, and agents must decide on whether to become an entrepreneur before they learn anything at all about their entrepreneurial skill. Hence there is need for a theoretical extension here.

Secondly, both types of skill are exogenous here. But more important, perhaps, the static nature of the model forces us to ignore the fact that entrepreneurial skills are surely developed through experience in wage-work: Individuals start businesses with the aid of experience that they have gained while working for firms run by other people. We still lack a theory that captures this important point, although the models of Chari and Hopenhayn (1991) and Holmes and Schmitz (1990) contain elements of it.

A third extension is to add leisure to the model, or a participation cost, so that participation will fluctuate in response to, say, Hicks-neutral technology shocks s_t that I analyzed at the outset of Section 5. This will allow a study of net flows over the business cycle between the three states: entrepreneurship, wage-work, and out of the labor force. Recent work by Benhabib et al. (1991) and Greenwood and Hercowitz (1991) looks at how the allocation of workers between the market and nonmarket sectors is affected by the business cycle, but these authors do not distinguish different types of market activity such as wage work and entrepreneurship. Moreover, these are representative agent models in which gross flows between the market and nonmarket sectors are the same as net flows. Ultimately, one wants a theory of both gross and net flows, as formulated, say, in Jovanovic and Moffit (1990) except in a context that has entrepreneurs.

Finally, income taxation and various subsidies to firm formation will impinge on the choice of whether to start one's own firm. Agents will compare after tax/subsidy expected earnings in making their decisions; we still lack a comprehensive study of which choices are on net subsidized, which are taxed, and some assessment of how big these taxes and subsidies are.

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Notes

[1] Related models are Kihlstrom and Laffont (1978) and Kanbur (1978). They incorporate risky income and risk aversion.

[2] Evans and Jovanovic (1989) assume that both entrepreneurial and working skills are heterogeneous, but they take the skill prices as exogenous. In Kihlstrom and Laffont (1979), neither labor skills nor managerial skills vary; selection into entrepreneurship depends only on people's attitudes towards risk. There is also a growing empirical literature on entrepreneurial selection, some of which is surveyed in Section 2 of Holtz-Eakin et al. (1992).

[3] The distribution of the ownershp of this capital stock will not matter to what follows and I will ignore it. Of course, if we were analyzing the distribution of income from all sources, we would have to specify this distribution as well.

[4] To see this take a total derivative of F_k, keeping $\mathrm{d}\ell = (\ell/k)\,\mathrm{d}k$ so that the capital labor ratio is constant. Since this will not change the value of F_k, we have

$$\left(F_{kk} + \frac{\ell}{k} F_{k\ell}\right) \mathrm{d}k = 0$$

for all $\mathrm{d}k$, and this implies the assertion.

[5] This assumption has recently been questioned by Rosen:

> It is difficult to even imagine technologies where it is only, say, the total strength of the workforce that matters rather than the strength of each worker and how it is distributed over workers employed. Linear combinability implies perfect substitution among worker types, whereas non-combinability restricts possible substitutions. This is partly due to indivisibilities, coordination, and joint production: the speed of an assembly line is constrained by the slowest worker. In cases such as these, the firm will desire a homogeneous labor force. (Rosen, 1983, p. 430)

[6] For the maximum to exist we need $Q'(h) < w$ for large enough h.

[7] For detailed evidence on the effect of observed personal characteristics on wage earnings and self-employed earnings, see Evans and Leighton (1989, p. 531, table 6). There is, however, some weak evidence that (Willis and Rosen, 1971, p. 529) the residual component of earnings of college educated people is negatively correlated with the unobservable component of the earnings of people with no college education. This is relevant because educated people are more likely to be self-employed (Evans and Leighton, 1989, p. 530, table 5).

[8] Some informational aspects of the two-skill Roy model when payoffs to skills as negatively correlated in the population, are analyzed in Jovanovic (1982a).

[9] Perhaps the two-period Lipman-Rummelt (1982) model of learning about one's own efficiency could be embedded in an overlapping generations general equilibrium economy.

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